

LATERAL VIBRATION AND INTERNAL FORCES OF GROUPED PILES IN LAYERED SOIL

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ABSTRACT: Simplified analytical models are developed for the lateral harmonic response of single piles and pile groups in layered soil. Pile-to-soil interplay is represented by a dynamic Winkler formulation based on frequency-dependent springs and dashpots. For pile-to-pile interaction, the wave field originating from each oscillating ("source") pile and the diffraction of this field by the adjacent ("receiver") piles are considered. The response of single piles and pile pairs is evaluated both numerically (through a transfer-matrix formulation) and analytically (introducing an efficient virtual-work approximation). Closed-form solutions are obtained: (1) for the impedance of single piles; (2) for the dynamic interaction factors between two piles; and (3) for the "additional" internal forces ("distress") developing in grouped piles because of pile-to-pile interaction, a phenomenon frequently ignored in current methods of analysis. Both swaying and rocking vibrational modes are considered. The effect of pile length and soil layering on the impedances and interaction factors is studied. The predictions of the model are in agreement with earlier results, while its simplicity offers a versatile alternative to rigorous solutions.

INTRODUCTION

Despite the significant progress in pile dynamics [see pertinent review articles by Roesset (1984), Novak (1991), Pender (1993), and Gazetas and Mylonakis (1998)], there is still a need for simple engineering procedures such as the code provisions developed for the seismic design of structures on spread footings (ATC-3 1978; NEHRP 1997). It is noted that rigorous solutions entail significant computational effort and are used primarily for research rather than as design tools (Blaney et al. 1976; Wolf and Von Arx 1978; Kuhlemeyer 1979; Kagawa and Kraft 1980; Kaynia and Kausel 1982; Sen et al. 1985). On the other hand, even approximate methods rely on sophisticated frequency-domain solutions involving extensive complex arithmetic and Bessel-type functions (Novak 1974; Nogami 1985).

The most suitable engineering method for calculating the dynamic interaction between pile and soil is the Winkler model in which the soil reaction to pile movement is represented by independent springs and dashpots distributed along the pile shaft. Although approximate, Winkler formulations are well accepted because (1) their predictions are in good agreement with results from more rigorous solutions (Novak 1974; Dobry et al. 1982; Gazetas and Dobry 1984; Pender 1993); (2) they can easily incorporate variation of soil properties with depth and with radial distance from the pile (Veletsos and Dotson 1986; Michaelides et al. 1998); and (3) they require smaller computational effort than finite and boundary element procedures.

A relatively simple method was proposed by Gazetas and Dobry (1984) for estimating the damping characteristics of horizontally loaded piles in layered soil. According to that method the "dashpot" coefficient $C(\omega)$ at the head of the pile is calculated through the following energy-conservation formula:

$$C(\omega) \approx \int_0^L c(z, \omega) Y(z)^2 dz \quad (1)$$

where $c(z, \omega)$ = distributed dashpots along the pile that account

for radiation and material energy dissipation in the soil; $Y(z)$ = pile deflection profile normalized to unit top amplitude; ω = circular frequency; and z = depth from the top. Gazetas and Dobry (1984) showed that results obtained with (1), even with an approximate $Y(z)$ profile and hand calculations, are in good agreement with more rigorous solutions.

The work reported here was motivated by the need to extend such simple engineering methods for pile dynamics. To this end, an analytical formulation is presented for estimating the following: (1) The complete dynamic impedance (i.e., both the stiffness and damping moduli) atop a pile; (2) the dynamic interaction factors between two adjacent piles (which are used for computing the impedance of pile groups); and (3) the dynamic internal forces (bending moments and shear forces) that develop in each pile of a group, in layered soil. To calculate these forces, the following two types of loading should be considered: (a) Forces from the vibrating superstructure transmitted onto the piles through the cap; and (b) additional dynamic loads ("distress") caused by pile-to-pile interaction. The latter loads are induced to piles in the form of dynamic tractions along their shafts imposed by the waves generated at the neighboring piles. Despite the fact that these additional forces have been recognized in some studies dealing with static loads (Randolph and Wroth 1979), their importance has remained to date unexplored.

SINGLE PILE

The lateral harmonic deflection $Y(z, t) = Y(z)\exp[i\omega t]$, of a vertical elastic pile embedded in a Winkler medium satisfies the following well-known equation:

$$\frac{d^4 Y(z)}{dz^4} + 4\lambda(z, \omega)^4 Y(z) = \frac{q(z)}{EI} \quad (2)$$

with $\lambda(z, \omega)$ given by

$$\lambda(z, \omega) = [(\ell(z, \omega) - m\omega^2)/4EI]^{1/4} \quad (3)$$

where EI and m = flexural rigidity and mass per unit length of the pile, respectively; $q(z)$ = distributed forces along the pile; $\ell(z, \omega) = k(z, \omega) + i\omega c(z, \omega)$ = complex soil impedance encompassing the stiffness, inertia, radiation, and hysteretic action of and in the soil; λ = a complex wavenumber associated with propagation of flexural waves along the pile (Wolf 1985).

In the special case of a pile of length L embedded in a homogeneous soil of thickness $h = L$, the solution to (2) yields, after some straightforward algebra and enforcement of boundary conditions, the following closed-form expressions for the complex

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impedances, $\mathcal{K}_{ij}(\omega) = K_{ij}(\omega) + i\omega C_{ij}(\omega)$, at the head of the pile (Mylonakis 1995):

$$\mathcal{K}_{hh} = 4EI\lambda^3 \frac{\sin 2\lambda h + \sinh 2\lambda h}{\pm 2 + \cos 2\lambda h + \cosh 2\lambda h} \quad (4a)$$

$$\mathcal{K}_{hr} = 2EI\lambda^2 \frac{-\cos 2\lambda h + \cosh 2\lambda h}{\pm 2 + \cos 2\lambda h + \cosh 2\lambda h} \quad (4b)$$

$$\mathcal{K}_{rr} = 2EI\lambda \frac{-\sin 2\lambda h + \sinh 2\lambda h}{\pm 2 + \cos 2\lambda h + \cosh 2\lambda h} \quad (4c)$$

where $\mathcal{K}_{hh}$, $\mathcal{K}_{rr}$, and $\mathcal{K}_{hr}$ are the swaying, rocking, and cross-swaying-rocking impedances, respectively. The plus sign in the denominator of (4) corresponds to the special case of a pile completely free of stresses at the tip, whereas the minus sign corresponds to a perfectly fixed base pile. Formulas for impedance functions for hinged piles at the tip are given in Mylonakis (1995).

For an infinitely long pile, the ratios on the right hand side of (4) tend to unity and thereby pile impedances converge to the well-known expressions (Pender 1993)

$$\mathcal{K}_{hh} = 4EI\lambda^3; \quad \mathcal{K}_{hr} = 2EI\lambda^2; \quad \mathcal{K}_{rr} = 2EI\lambda \quad (5a-c)$$

For an arbitrary variation of soil stiffness with depth, no exact solution to (2) has been derived [to the writers' knowledge, the only exact solutions to (2) involving variable soil properties are those of Hetenyi (1946) and Franklin and Scott (1979), both derived for $k(z)$ increasing proportionally with depth, under static conditions. These solutions are expressed in terms of power series, which complicates their routine engineering use]. In this paper, an efficient approximate solution, capable of handling vertically inhomogeneous soils, is developed through energy considerations. To this end, the unknown solution $Y(z)$ in (2) is replaced by an approximate function $\chi(z)$. On multiplying by a second function $\phi(z)$ and after successive integrations by parts over the pile length (Banerjee 1994), the impedance of the pile is obtained as

$$\mathcal{K}_{ij}(\omega) \approx EI \int_0^L \left[\frac{d^2\chi(z)}{dz^2} \frac{d^2\phi(z)}{dz^2} + 4\lambda(z, \omega)^4 \chi(z)\phi(z) \right] dz \quad (6)$$

Noting that the foregoing virtual-work expression is not sensitive to the details of the selected shape functions (Clough and Penzien 1993), for long piles $\chi(z)$ and $\phi(z)$ can be taken equal to the deflected shape of a long pile in homogeneous soil

$$\chi(z) = e^{-\mu z}(\sin \mu z + \cos \mu z) \quad (7a)$$

and

$$\phi(z) = \frac{e^{-\mu z}}{\mu} \sin \mu z \quad (7b)$$

Of these functions, $\chi(z)$ is the pile deflection shape caused by a unit imposed head displacement (under zero rotation), whereas $\phi(z)$ is the deflected shape caused by a unit head rotation (under zero displacement); μ is a "shape" parameter, which, for homogeneous soil, is the wavenumber λ of (3). In nonhomogeneous soil μ can be approximated with the mean value of λ within the active length L_a of the pile

$$\mu(\omega) \approx \frac{1}{L_a} \int_0^{L_a} \lambda(z, \omega) dz \quad (8)$$

Note that L_a is the critical pile length beyond which the pile behaves as an infinitely long beam; i.e., an increase in pile length does not change the stiffness atop the pile. Typically, L_a is of the order of 10–15 pile diameters (Velez et al. 1983).

For a soil consisting of discrete homogeneous layers the

integrals in (6) can be evaluated analytically. As an example, for a long (i.e., $L > L_a$) pile penetrating a soil of two layers, (6)–(8) yield

$$\mathcal{K}_{hh} = \frac{EI}{\mu} [e^{-2\mu h_1}(\lambda_2^4 - \lambda_1^4)(\sin 2\mu h_1 + \cos 2\mu h_1 + 2) + \mu^4 + 3\lambda_1^4] \quad (9a)$$

$$\mathcal{K}_{rr} = \frac{EI}{2\mu^3} [e^{-2\mu h_1}(\lambda_2^4 - \lambda_1^4)(\sin 2\mu h_1 - \cos 2\mu h_1 + 2) + 3\mu^4 + \lambda_1^4] \quad (9b)$$

$$\mathcal{K}_{hr} = \frac{EI}{\mu^2} [e^{-2\mu h_1}(\lambda_2^4 - \lambda_1^4)(\sin 2\mu h_1 + 1) + \mu^4 + \lambda_1^4] \quad (9c)$$

with

$$\mu = \left[\frac{\lambda_1 h_1 + \lambda_2 (L_a - h_1)}{L_a} \right] \quad (9d)$$

where h_1 = thickness of the surface layer. Eq. (9d) holds for L_a larger than h_1 . If the surface layer goes deeper than the "active" pile length, μ becomes equal to λ_1 . In such a case, the exponentials on the right hand side of (9a)–(9d) tend to zero and the foregoing expressions converge to those of the infinitely long pile in homogeneous soil: $\mathcal{K}_{hh} = 4EI\lambda_1^3$; $\mathcal{K}_{hr} = 2EI\lambda_1^2$; $\mathcal{K}_{rr} = 2EI\lambda_1$.

The developed method can incorporate any type of soil springs and dashpots, for static (Lam and Martin 1986; Reese 1986; Norris 1994) or dynamic conditions (Novak et al. 1978; Veletsos and Dotson 1986). In this work we utilize the finite-element-based springs and dashpots of Makris and Gazetas (1992). Also, analytical expressions for the active pile length can be obtained from Velez et al. (1983), Pender (1993), Gazetas (1991), among others. The latter is utilized here.

Static pile stiffness as predicted by (4a)–(4c) is graphed in Fig. 1. The plotted values are normalized by the corresponding stiffness of the infinitely long pile. Several interesting trends are worthy of note. First, the horizontal stiffness of a floating

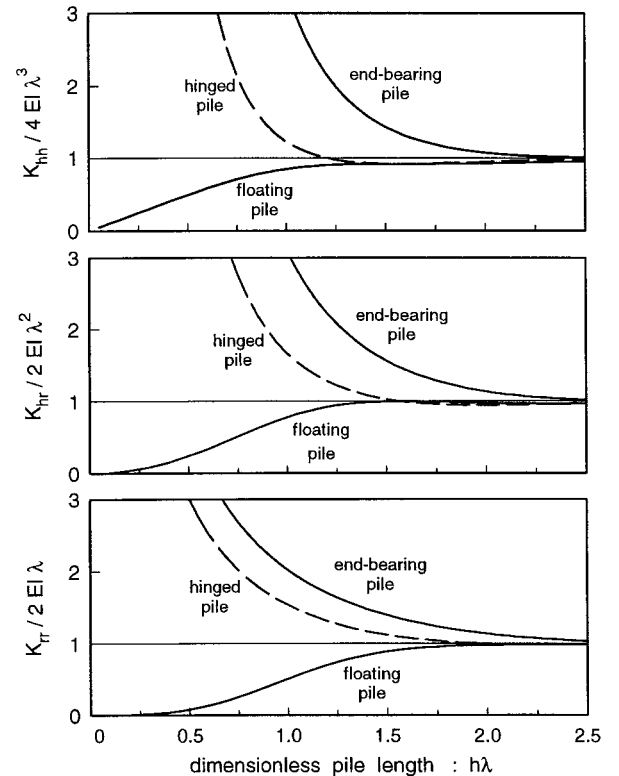


FIG. 1. Variation with Pile Length of Static Stiffness of Single Piles for Various Boundary Conditions at Pile Tip [See (4)]

pile increases almost linearly with pile length up to a value of about $h\lambda \approx 0.50$. This implies that within the range $0 < h\lambda < 0.50$ the pile behaves as essentially rigid. For larger pile lengths, the pile becomes gradually more flexible, its stiffness reaching 90% that of the infinitely long pile at $h\lambda \approx 1.50$. Second, with end-bearing piles, the horizontal stiffness decreases monotonically with pile length, reaching a minimum value at $h\lambda$ about 2.3. Third, for "hinged-tip" piles, an interesting phenomenon is observed: at $h\lambda$ about 0.50 and 2.0 the horizontal and cross-stiffnesses become increasing functions of pile length. Mylonakis (1995) showed that with increasing pile length K_{hr} undulates exhibiting peaks at $h\lambda_{(k)} = k\pi/2$, [where $k = \text{positive integer}$]. The first peak obtained by this formula, $h\lambda_{(1)} = \pi/2 = 1.57$, is in agreement with that seen in Fig. 1.

Results from (9) are plotted in Fig. 2 for a pile in a two-layered soil. The solutions are compared with: (1) a complete ("exact") Winkler analysis, (2) an approximate Winkler analysis using a homogeneous soil with average properties, and (3) a formula fitted to finite-element results by Dobry et al. (1982). Although the predictions of the (9a)–(9d) are in excellent agreement with the more rigorous results, the assumption of equivalent homogeneous soil overpredicts pile stiffness. The deviation is larger in the horizontal component—which is controlled by the soil reaction close to the surface—as opposed to rocking, which mobilizes soil reaction at greater depths and for which the assumption of equivalent homogeneous soil is more accurate.

Pise (1982) calculated the static stiffness of single piles in two-layer soil by integrating Mindlin's equation in the approximate manner proposed by Poulos (Poulos and Davis 1980). A comparison against his results is depicted in Fig. 3, referring to a stiff free-head pile. In the case of free-head piles (i.e., piles carrying zero moment at the top), the horizontal pile impedance is calculated as $\mathcal{H}_h = \mathcal{H}_{rr}/(\mathcal{H}_{hh}\mathcal{H}_{rr} - \mathcal{H}_{hr}^2)$. The

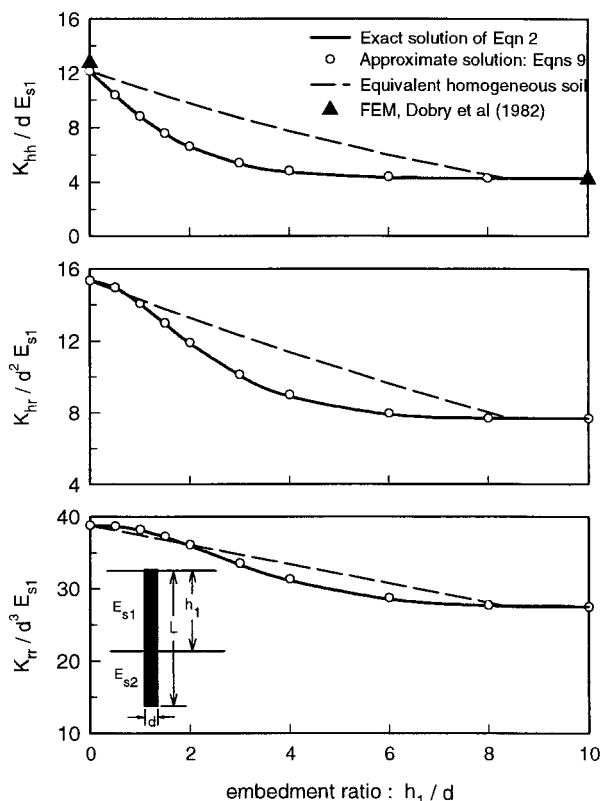


FIG. 2. Single Pile in Two-Layered Soil (Static Conditions). Comparison of Derived Expression in (9) with Results from Alternative Analyses; $E_p/E_{s1} = 1,000$, $L/d = 20$, $E_{s1}/E_{s2} = 1/4$; [Modulus of Equivalent Homogeneous Soil was Computed as $E = E_{s1}h_1/L_a + E_{s2}(1 - h_2/L_a)$, L_a Being the Active Length of Pile]

agreement between the two methods is satisfactory for the whole range of pile embedment ratios.

Eqs. (3), (7), and (8) imply that the deflected shape of a harmonically vibrating pile is a rather complicated function of depth, associated with the complex wavenumber $\mu(\omega)$. However, Dobry et al. (1982) and Gazetas and Dobry (1984) showed that the deflected shape of a pile is little affected by frequency, and that reasonable estimates of the pile impedance can be made by considering $\chi(z)$ and $\phi(z)$ to be equal to their static values. Based on this approximation the following simplification is possible: the real and imaginary parts in (6) can be separated, and thus, the spring and the dashpot coefficients at the pile head can be calculated separately, without using complex arithmetic.

As an example, the case of a pile embedded in a two-layered soil is presented here: the stiffness K_{ij} is obtained from (9a)–(9d), simply by replacing the complex quantities λ_1^4 and λ_2^4 with their real parts $(k_1 - m\omega^2)/4EI$ and $(k_2 - m\omega^2)/4EI$, respectively. The corresponding damping coefficients are obtained as

$$C_{hh} = \frac{1}{4\mu} [e^{-2\mu h_1}(c_2 - c_1)(\sin 2\mu h_1 + \cos 2\mu h_1 + 2) + 3c_1] \quad (10a)$$

$$C_{rr} = \frac{1}{8\mu^3} [e^{-2\mu h_1}(c_2 - c_1)(\sin 2\mu h_1 - \cos 2\mu h_1 + 2) + c_1] \quad (10b)$$

$$C_{hr} = \frac{1}{4\mu^2} [e^{-2\mu h_1}(c_2 - c_1)(\sin 2\mu h_1 + 1) + c_1] \quad (10c)$$

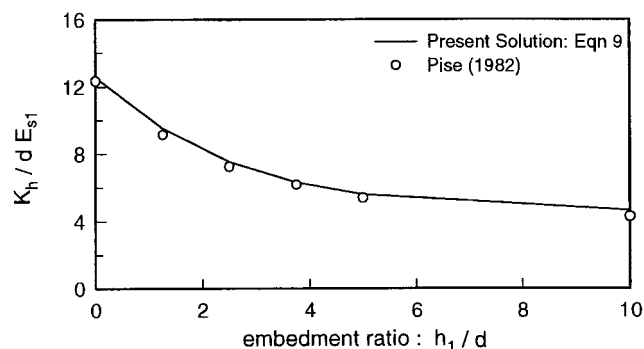


FIG. 3. Single Free-Head Pile in Two-Layered Soil; $L/d = 25$, $E_p/E_{s1} = 31,800$, $E_{s2}/E_{s1} = 4$, $\nu_s = 0.5$. Comparison of Derived Equation in (9) with Numerical Solution of Pise (1982)

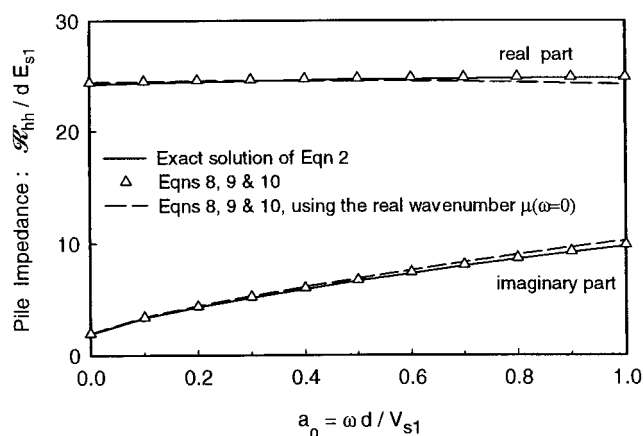


FIG. 4. Horizontal Impedance of Single Pile in Two-Layered Soil. Comparison of Derived Equations in (9) and (10) with Exact Solution of (2); $L/d = 20$, $E_p/E_{s1} = 875$; $E_{s2}/E_{s1} = 21$; $h_1/d = 1.5$; $\nu_s = 0.4$; $p_p/p_s = 1.5$; $\beta = 5\%$

where c_1 and c_2 = moduli of the distributed dashpots in the first and second layer, respectively.

The validity of the foregoing approximation is illustrated in Fig. 4 where the horizontal impedance of a pile in a two-layer soil is plotted, based on results from (a) an exact solution of (2), (b) the present solution with the complex wavenumbers μ [see (10), (9), (8), and (3)], and (c) the simplified version of the present solution using the static wave-number (real number) μ ($\omega = 0$). The agreement between the three approaches is quite satisfactory in the whole range of frequencies $0 < a_0 < 1$ ($a_0 = \omega d/V_s$), confirming the validity of the approximations.

INTERACTION BETWEEN TWO PILES

In addition to loading transmitted to piles from the superstructure through the cap, grouped piles experience additional dynamic loading imposed along their shafts by waves emitted from the neighboring piles. Such dynamic group effects can be treated approximately using complex interaction factors, which account for the dynamic interplay between two piles. This type of analysis, referred to as the “superposition method,” provides good (although approximate) estimates of the dynamic response of a pile group (Kaynia and Kausel 1982; Dobry and Gazetas 1988; El-Marsafawi et al. 1992).

The interaction factor α between two piles is defined based on the response of a pile carrying no load at its head (hereafter called pile 2 or “receiver” pile), subjected to the ground vibrations produced by a neighboring pile (hereafter called pile 1 or “source” pile), which is loaded with either (1) a horizontal force or (2) a moment. The interaction factor then is defined as the response (translation or rotation) atop the receiver pile, normalized by the corresponding response of the source pile caused by its own loading (Poulos and Davis 1980).

In flexural vibrations the interaction factor is expressed by a 2×2 complex matrix

$$[\alpha] = [\alpha(s, \theta)] = \begin{bmatrix} \alpha_{uP} & \alpha_{uM} \\ \alpha_{\phi P} & \alpha_{\phi M} \end{bmatrix} \quad (11)$$

where α_{uP} = swaying interaction factor; $\alpha_{\phi M}$ = rocking interaction factor; α_{uM} and $\alpha_{\phi P}$ = cross-swaying-rocking factors; s = axis-to-axis distance between the piles; and θ = “aperture” angle between the direction of loading and the line connecting the pile centers.

Dobry and Gazetas (1988) proposed a simple model for calculating the dynamic interaction factor between piles in homogeneous soil. According to that model, the response of the receiver pile to the oscillations of the source pile is equal approximately to the response of the free-field soil at the location of the receiver pile. For this to be true, Dobry and Gazetas (1988) assume that: (1) cylindrical waves are generated along the source pile, emitted simultaneously from all points along its shaft. The waves propagate through the soil and “strike” simultaneously the shaft of the receiver pile; (2) the receiver pile follows exactly this attenuated ground motion (this implies that possible interaction between the receiver pile and the surrounding soil is neglected); and (3) the rocking and cross-swaying-rocking interaction factors are negligibly small.

Based on the foregoing assumptions, the interaction factors are written as

$$\alpha_{uP}(s, \theta) \approx \psi(s, \theta); \quad \alpha_{uM} \approx \alpha_{\phi P} \approx \alpha_{\phi M} \approx 0 \quad (12a,b)$$

where $\psi(s, \theta)$ = attenuation function of the horizontal soil displacement with radial distance from the pile and direction of loading

$$\psi(s, \theta) \equiv \frac{U(s, \theta, z)}{U(d/2, \theta, z)} \approx \psi(s, 0) \cos^2 \theta + \psi(s, \pi/2) \sin^2 \theta \quad (13a)$$

$$\psi(s, 0) \approx \left(\frac{2s}{d} \right)^{-1/2} \exp \left[-(\beta_s + i) \left(\frac{s}{d} - \frac{1}{2} \right) \frac{V_s}{V_{La}} a_0 \right] \quad (13b)$$

$$\psi \left(s, \frac{\pi}{2} \right) \approx \left(\frac{2s}{d} \right)^{-1/2} \exp \left[-(\beta_s + i) \left(\frac{s}{d} - \frac{1}{2} \right) a_0 \right] \quad (13c)$$

where $U(s, \theta, z)$ = horizontal soil displacement; $\psi(s, 0)$ and $\psi(s, \pi/2)$ = attenuation functions corresponding to waves traveling along and perpendicular to the direction of loading, respectively; V_{La} is the so-called “Lysmer’s analogue” wave velocity $V_{La} = 3.4V_s/[1 - \nu]\pi]$ (Gazetas and Dobry 1984); β_s = hysteretic soil damping; and $a_0 = \omega d/V_s$.

Despite the simplicity and the approximations, the Dobry and Gazetas (1988) method provides very accurate results for stiff piles in homogeneous soil (Novak 1991; Wolf et al. 1992). In fact, the effectiveness of the model is rather surprising because several problem parameters (e.g., pile-soil stiffness contrast E_p/E_s) are not included in (12) and (13). The accuracy of the method gradually deteriorates when dealing with strongly inhomogeneous soil or piles of small slenderness (Mylonakis 1995).

New Proposed Model for Lateral Pile-to-Pile Interaction

To overcome the limitations of the Dobry and Gazetas (1988) method, an improved model is developed here. With reference to the two-pile system of Fig. 5, the proposed approximate method involves the following three consecutive steps.

Step 1. The deflected shape of the source pile, hereafter referred to as $Y_{1i}(z)$, to a unit force or moment applied at its head is determined using any pertinent analytical approach.

Step 2. Cylindrical waves are emitted from the periphery of the source pile with amplitude equal to the deflected pile shape $Y_{1i}(z)$. With the soil composed by distinct homogeneous horizontal layers, it is assumed that the waves propagate in an essentially horizontal manner within each individual layer. This implies that the radial spreading of these waves, although different for each layer, still obeys (even if approximately) the plane-strain attenuation law expressed by $\psi(s, \theta)$. Therefore, in the soil layer i , the free-field soil displacement at a distance s and angle θ from the direction of loading, is given by

$$U(s, z, \theta)_i \approx \psi(s, \theta) Y_{1i}(z)_i \quad (14)$$

where i = number of the layer. (This essentially is a “separation of variables” approximation.)

Step 3. The receiver pile does not follow exactly the free-field motion of step 2. Its inertial and flexural resistance would give rise to an interaction between this (the receiver) pile and the surrounding soil, leading to a diffraction of the arriving wave field. Thereby, the pile displacement will be different than that given by (14). Moreover, a rotation is generated at the head of the receiver pile, which cannot be calculated directly from (14). To account in a simple yet realistic way for this interaction, the receiver pile is modeled as a Winkler-supported beam in which the excitation $U(s, z)$ is applied at the support of the distributed springs and dashpots attached to the pile. The mechanics of this loading is in a sense the reverse of step 1. In step 1 the source pile induces displacements on soil through its “reacting” springs-dashpots, whereas in step 3 the soil induces displacements on the receiver pile through its “transmitting” springs-dashpots. A similar three-step model was proposed by Makris and Gazetas (1992) who, however, considered only infinitely-long fixed-head piles in homogeneous soil, and by Mylonakis and Gazetas (1998a,b) who studied the vertical response mode. In this work we analyze lateral vibrations considering both swaying and rocking and accounting for finite pile length and soil layering.

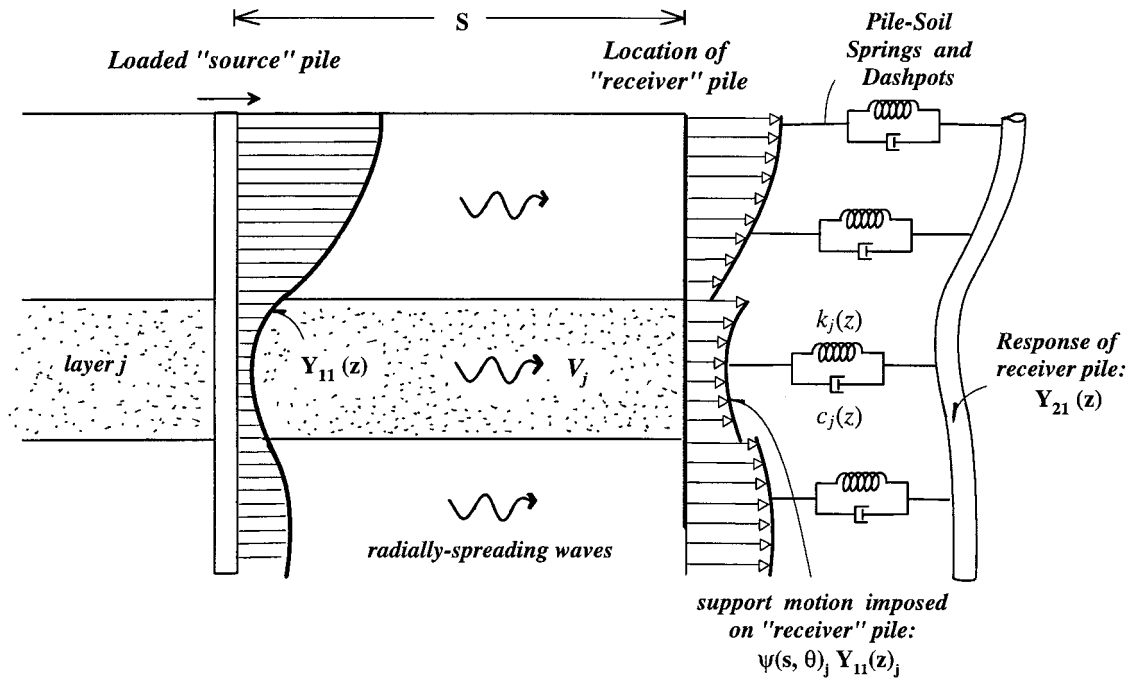


FIG. 5. Schematic Illustration of Proposed Model for Computing Influence of Head-Loaded Source Pile on Adjacent Receiver Pile Carrying No Load at Its Head, in Layered Soil

For a receiver pile, the dynamic equilibrium of an infinitesimal pile segment produces the following equation governing the deflection $Y_{21}(z)$ of the pile (Mylonakis 1995):

$$\frac{d^4 Y_{21}(z)}{dz^4} + 4\lambda^4 Y_{21}(z) = k(z, \omega) \psi(s, \theta) Y_{11}(z) / EI \quad (15a)$$

The forcing term on the right side of (15a) is equal to the attenuated free-field soil displacement times the dynamic soil impedance, divided by the flexural rigidity of the pile; $Y_{11}(z)$ is the deflected shape of the source pile, obtained for each particular soil layer from the solution of the homogeneous part of (2)

$$Y_{11}(z) = \exp(\lambda z)(A_{11} \sin \lambda z + B_{11} \cos \lambda z) + \exp(-\lambda z)(C_{11} \sin \lambda z + D_{11} \cos \lambda z) \quad (15b)$$

where A_{11} , B_{11} , C_{11} , and D_{11} = integration constants determined from the boundary conditions of the source pile (step 1). The solution to (15a) is

$$Y_{21}(z) = \frac{k(\omega)}{k(\omega) - m\omega^2} \psi(s, \theta) \frac{\lambda z}{4} Y_{11}(z) + \exp(\lambda z)(A_{21} \sin \lambda z + B_{21} \cos \lambda z) + \exp(-\lambda z)(C_{21} \sin \lambda z + D_{21} \cos \lambda z) \quad (16)$$

where A_{21} , B_{21} , C_{21} , and D_{21} = integration constants to be determined from the boundary conditions of the receiver pile (i.e., zero shear force and bending moment at the pile head and continuity of force, moment, displacement, and rotation at the various interfaces). Differentiating (16) with respect to z yields the flexural rotations, $\Theta_{21}(z)$, along the receiver pile. The interaction factors are calculated as: $\alpha_{uP} = Y_{21}(0)/Y_{11}(0)$, $\alpha_{\phi P} = \Theta_{21}(0)/\Theta_{11}(0)$, $\alpha_{uM} = Y_{21}(0)/Y_{11}(0)$, $\alpha_{\phi M} = \Theta_{21}(0)/\Theta_{11}(0)$.

As a first application, the interaction between long piles in homogeneous soil is examined. In this case, the constants A_{11} , B_{11} , A_{21} , and B_{21} vanish ensuring finite response at large depths down the pile. The interaction factors can be obtained in closed form and can be written as a product of two complex-valued functions (Mylonakis 1995)

$$\begin{bmatrix} \alpha_{uP} & \alpha_{uM} \\ \alpha_{\phi P} & \alpha_{\phi M} \end{bmatrix} = \psi(s, \theta) \begin{bmatrix} \zeta_{uP} & \zeta_{uM} \\ \zeta_{\phi P} & \zeta_{\phi M} \end{bmatrix} \quad (17)$$

where ζ_{uP} , ζ_{uM} , $\zeta_{\phi P}$, and $\zeta_{\phi M}$ are given by

$$\zeta_{uP} = \frac{3}{4} \frac{k(\omega)}{k(\omega) - m\omega^2} \quad (18a)$$

$$\zeta_{uM} = \zeta_{\phi P} = \frac{1}{2} \frac{k(\omega)}{k(\omega) - m\omega^2} \quad (18b)$$

$$\zeta_{\phi M} = \frac{1}{4} \frac{k(\omega)}{k(\omega) - m\omega^2} \quad (18c)$$

where ψ = induced free-field displacement; and ζ are factors accounting for the effect of diffraction of the arriving wave field caused by the flexural rigidity and inertia of the receiver pile and the interaction between pile and surrounding soil. [It is noted that Dobry and Gazetas (1988) neglected the interaction between receiver pile and soil, implicitly assuming $\zeta_{uP} = 1$, $\zeta_{uM} = \zeta_{\phi P} = \zeta_{\phi M} = 0$.]

Several interesting features are worthy of note: first, with decreasing frequency, ζ_{uP} approaches 3/4 quickly, which implies that the response of the head of the receiver pile is equal to only 75% of the free-field motion—as opposed to 100% for the Dobry and Gazetas (1988) simple solution; second, at high frequencies the term $(-m\omega^2)$ in (18a)–(18c) dominates (which implies that the receiver pile resists the induced motion mainly by its inertia), and thereby, all interaction factors tend to zero; third, the cross-interaction factors $\zeta_{\phi P}$ and $\zeta_{\phi M}$ are equal, hence the matrix $[\alpha]$ is symmetric.

In the case of piles of finite length, the interaction factors still can be obtained analytically. For example, for piles floating in a homogeneous half-space the corresponding diffraction functions ζ_{uP} , $\zeta_{\phi M}$, and ζ_{uM} can be written as

$$\begin{bmatrix} \zeta_{uP} & \zeta_{uM} \\ \zeta_{\phi P} & \zeta_{\phi M} \end{bmatrix} = \begin{bmatrix} f_{hh} & f_{hr} \\ f_{rh} & f_{rr} \end{bmatrix} \begin{bmatrix} \mathcal{H}_{hh} & \mathcal{H}_{hr} \\ \mathcal{H}_{rh} & \mathcal{H}_{rr} \end{bmatrix}^{-1} \quad (19a)$$

where $\mathcal{H}_{hh}$, $\mathcal{H}_{hr}$, $\mathcal{H}_{rh}$, and $\mathcal{H}_{rr}$ are the impedances of a single solitary pile [see (4a)–(4c)]; the functions f_{hh} , f_{hr} , f_{rh} , and f_{rr} are given by (Mylonakis 1995)

$$f_{hh} = \frac{k(\omega)}{k(\omega) - m\omega^2} \cdot \frac{1/2(C_4 + H_4 - 4) + C_2T_2 + \lambda h[H_2(1 + C_2) - S_2(1 + T_2)]}{(C_2 + T_2)^2 - 4} \quad (19b)$$

$$f_{hr} = \frac{k(\omega)}{k(\omega) - m\omega^2} \frac{1}{4\lambda} \cdot \frac{4\lambda h(C_2H_2 - 1) - T_2S_2 - C_2H_2 + 1/2(H_4 + S_4)}{(C_2 + T_2)^2 - 4} \quad (19c)$$

$$f_{rh} = \frac{k(\omega)}{k(\omega) - m\omega^2} \frac{-\lambda}{2} \frac{T_2S_2 - 1/2S_4 - C_2H_2 - 2\lambda hS_2H_2 + 1/4H_4}{(C_2 + T_2)^2 - 4} \quad (19d)$$

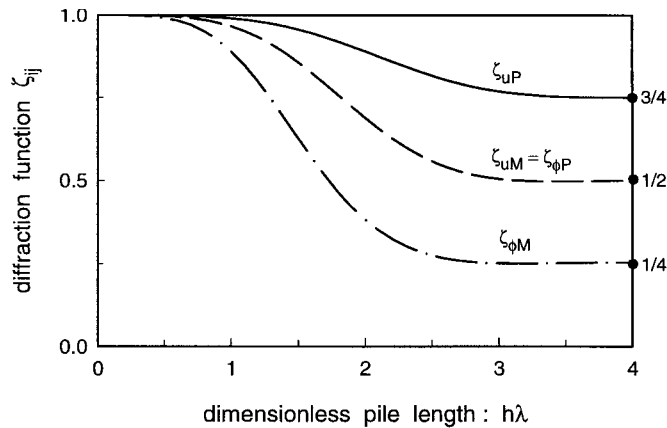


FIG. 6. Effect of Pile Length on Diffraction Functions ζ_{ij} for Pile in Homogeneous Soil

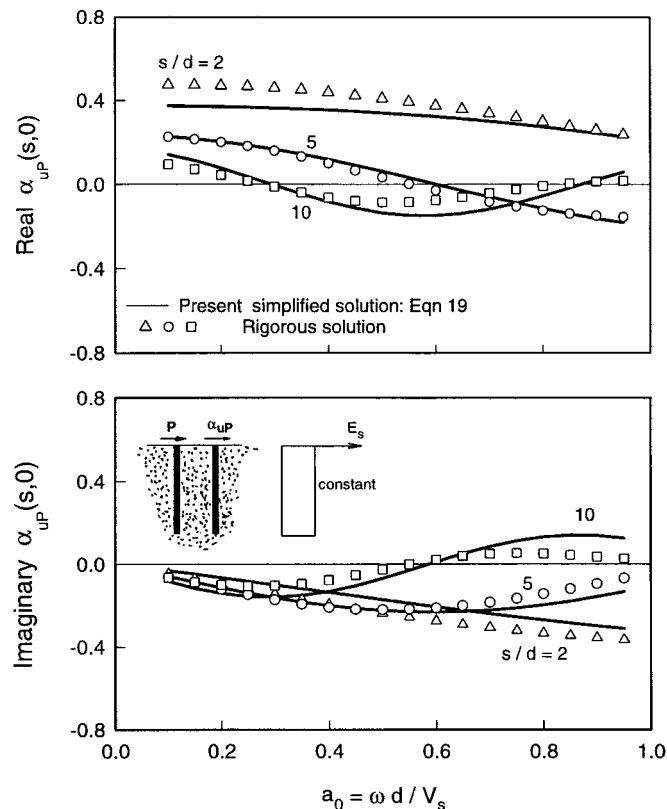


FIG. 7. Lateral Interaction Factors between Two Piles in Homogeneous Soil for Various Pile Separations s/d ($L/d = 20$; $E_p/E_s = 1,000$; $\nu_s = 0.4$; $\rho_p/\rho_s = 1.5$; $\beta_s = 5\%$). Comparison of Proposed Method with the Rigorous Method of Kaynia and Kausel (1982)

$$f_{rr} = \frac{k(\omega)}{k(\omega) - m\omega^2} \frac{2(T_1S_1 + C_1H_1)(-T_1S_1 + C_1H_1 + 2\lambda hS_1H_1)}{(C_2 + T_2)^2 - 4} \quad (19e)$$

where S_n , C_n , H_n , and T_n denote the functions $\sin(n \times \lambda h)$, $\cos(n \times \lambda h)$, $\sinh(n \times \lambda h)$, and $\cosh(n \times \lambda h)$, respectively. Note that, with increasing pile length (19a)–(19e) reduce to (18a)–(18c).

Fig. 6 plots ζ_{uP} , $\zeta_{\phi M}$, and ζ_{uM} for piles of finite length in homogeneous soil under static conditions. For zero pile length all diffraction functions tend to unity, which is expected because a floating pile of zero length has no flexural resistance and follows exactly the free-field motion. For $h\lambda > 0$, the functions decrease monotonically with pile length, reaching their asymptotic values $\zeta_{uP} = 3/4$, $\zeta_{uM} = 1/2$, and $\zeta_{\phi M} = 1/4$, at $h\lambda \approx 3.0$, which is larger than the active length value of the corresponding solitary piles (Fig. 1). This implies that two interacting piles mobilize soil reaction at greater depths than a solitary pile.

In Figs. 7–9, interaction factors α_{uP} , $\alpha_{\phi M}$, and α_{uM} for piles in homogeneous soil are plotted against results obtained by the writers using the rigorous formulation of Kaynia and Kausel (1982). Notice the oscillatory behavior of the interaction factors with frequency. The performance of the simplified method is evident in the whole range of frequencies and pile separations examined.

From (18a)–(18c) the following relations can be derived between the interaction factors for infinitely-long piles:

$$\alpha_{\phi P} = \frac{2}{3} \alpha_{uP}; \quad \alpha_{\phi M} = \frac{1}{3} \alpha_{uP} \quad (20a,b)$$

Comparison of the foregoing expressions with the empirical static relations of Randolph (1981)

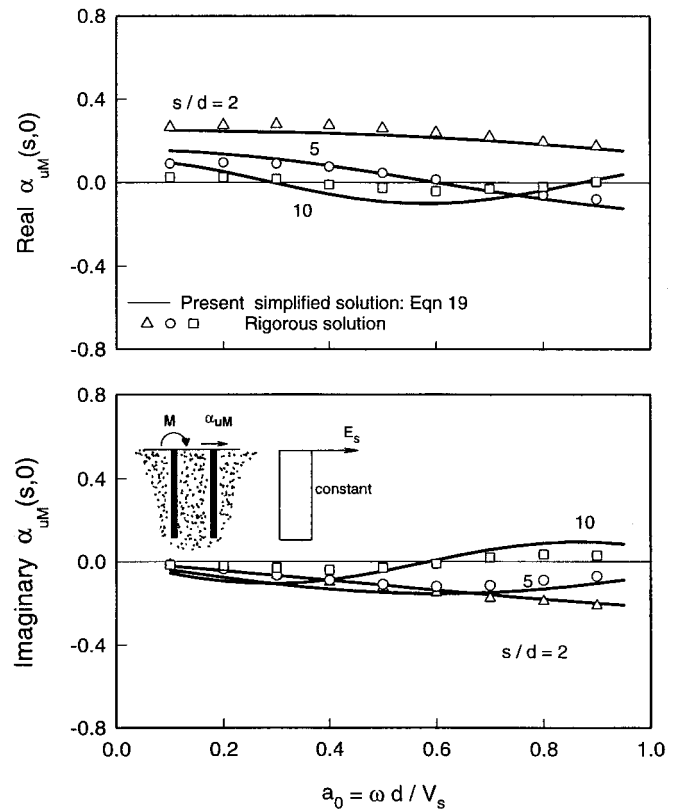


FIG. 8. Cross-Swaying-Rocking Interaction Factor between Two Piles in Homogeneous Soil for Various Pile Separations s/d ($L/d = 20$; $E_p/E_s = 1,000$; $\nu_s = 0.4$; $\rho_p/\rho_s = 1.5$; $\beta_s = 5\%$). Comparison of Proposed Method with the Rigorous Method of Kaynia and Kausel (1982)

$$\alpha_{\phi P} = (\alpha_{uP})^2; \quad \alpha_{\phi M} = (\alpha_{uP})^3 \quad (21a,b)$$

is performed graphically in Fig. 10. Also plotted in the graph are the numerical results from Figs. 7–9 obtained with the rigorous method of Kaynia and Kausel (1982) for nine different frequencies and three pile separations. At low frequencies ($a_0 = 0.1$), Randolph's curves fit better to the rigorous results. With increasing frequency, the rigorous results move closer to the derived expressions in (20). To explain why, recall that with increasing frequency, waves emitted from a pile tend to follow horizontal paths—a result of wave interference (Gazetas 1987), in accord with the assumption of this study (Fig. 5). In that sense, the derived expressions in (20) are asymptotically “correct” at high frequencies. Nevertheless, because the magnitude of the rocking and cross-interaction factors is quite small, both sets of expressions [i.e., (20) and (21)] provide acceptable engineering estimates.

For multilayer soil, interaction factors can be calculated by repeating (16) to all different layers. The integration constants for each layer A_{21} , B_{21} , C_{21} , and D_{21} can be eliminated by enforcing continuity of forces and displacements at the various interfaces. This is done efficiently with a layer transfer-matrix formulation (Haskell-Thomson method). Details can be found elsewhere (Mylonakis 1995).

Fig. 11 refers to a pair of interacting piles in a two-layered soil profile. The soil consists of a surface layer over a stiffer homogeneous half-space. It is apparent that the presence of the soft layer reduces the values of the interaction factors compared with the homogeneous soil (Fig. 7), in agreement with the rigorous solution.

Fig. 12 refers to an inhomogeneous half-space with Young's modulus increasing linearly with depth, from $E_s = E_p/1,250$ at the surface to $E_p/500$ at the pile tip (located at depth $L = 20d$). To analyze pile-to-pile interaction, the soil profile was discre-

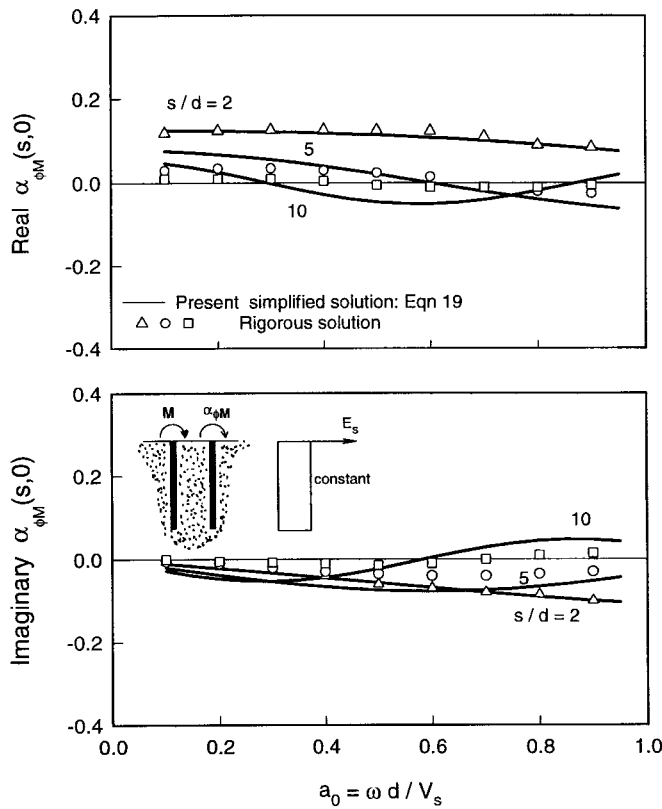


FIG. 9. Rocking Interaction Factor between Two Piles in Homogeneous Soil for Various Pile Separations s/d ($L/d = 20$; $E_p/E_s = 1,000$; $\nu_s = 0.4$; $\rho_p/\rho_s = 1.5$; $\beta_s = 5\%$). Comparison of Proposed Method with the Rigorous Method of Kaynia and Kausel (1982)

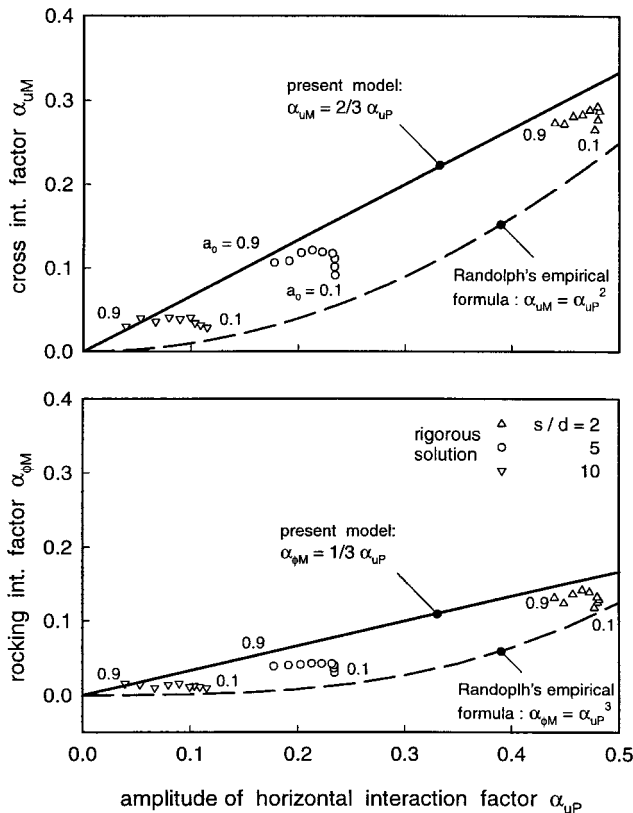


FIG. 10. Relations between three Interaction Factors for Long Piles in Homogeneous Soil. Comparison of Proposed Method With: (1) Empirical Formulas of Randolph (1981); (2) Results using the Rigorous Method of Kaynia and Kausel (1982)

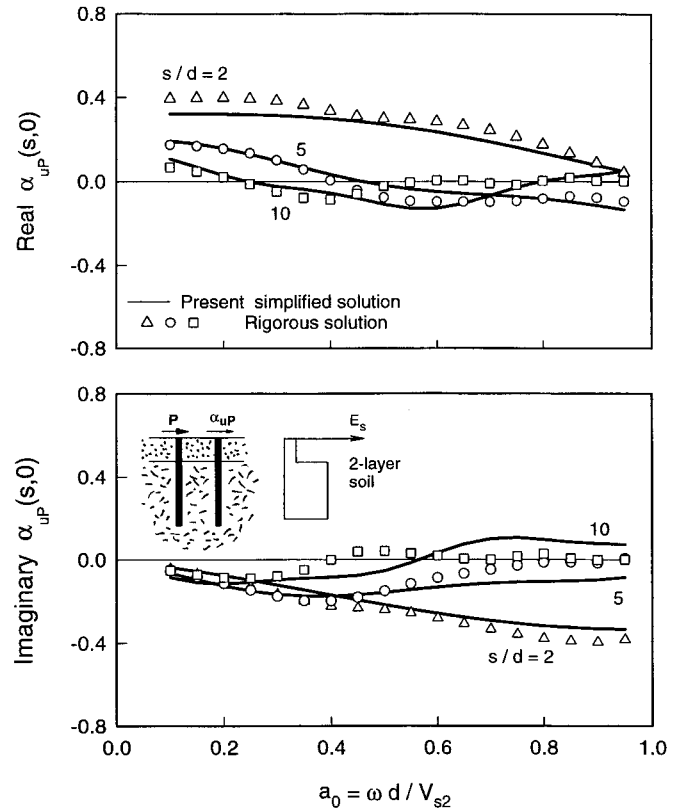


FIG. 11. Lateral Interaction Factor between Two Piles in Half-Space Overlain by Soft Homogeneous Layer with $E_p/E_{s1} = 10,000$ and $h_1/d = 1$ ($L/d = 20$; $E_p/E_{s2} = 1,000$; $\nu_s = 0.4$; $\rho_p/\rho_s = 1.3$; $\beta_s = 5\%$). Comparison of Proposed Method with the Rigorous Method of Kaynia and Kausel (1982)

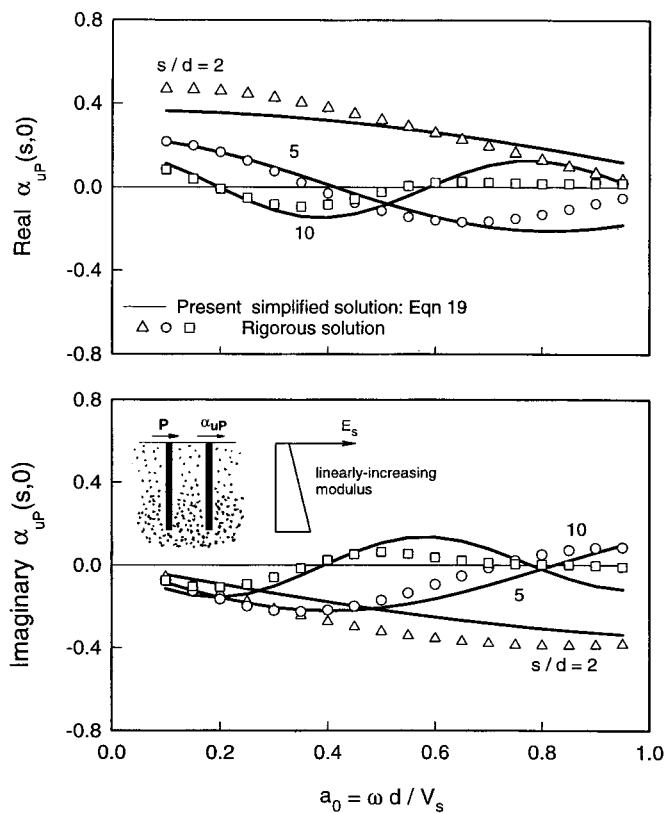


FIG. 12. Lateral Interaction Factors between Two Piles in Inhomogeneous Soil with Linearly Increasing Modulus, from $E_s = E_p/1,250$ at Surface to $E_s = E_p/500$ at $z = L$ ($L/d = 20$; $\nu_s = 0.4$; $\rho_p/\rho_s = 1.3$; $\beta_s = 5\%$). Comparison of Proposed Method with the Rigorous Method of Kaynia and Kausel (1982)

tized in eight piecewise homogeneous layers. Despite the fact that the simplifying assumption of purely horizontal wave propagation may in principle be less accurate in this case, the accord with the rigorous results is satisfactory.

Response of Pile Groups

With the superposition method the dynamic impedance of a group of m identical piles connected through a rigid cap is calculated by superimposing the interactions factors between individual pile pairs. The method is well known (Poulos and Davis 1980) and only results are presented here.

In Fig. 13, the horizontal dynamic impedance of a 3×3 pile group is plotted in terms of the so-called group efficiency factor (defined as the dynamic impedance of the pile group divided by the sum of the individual static stiffnesses of the piles). The pile group is embedded in a homogeneous soil profile. At low (static) frequencies, the real part of the group factor reduces to the familiar static efficiency factor, which, for elastic pile-soil systems, is always smaller than unity. At relatively low frequencies, the wavelengths emitted by the piles are much larger than the distances between piles; thereby the soil mass within the group tends to vibrate in-phase with the piles. However, beyond a certain frequency level wave-interference phenomena become apparent leading to efficiency factors that far exceed unity. The proposed method compares well with corresponding results obtained using the rigorous method of Kaynia and Kausel (1982).

The significance of soil inhomogeneity on the impedance of a 3×3 pile group is illustrated in Fig. 14 for the soil profile of Fig. 12. The figure contrasts the simplified method proposed here and the rigorous solution of Kaynia and Kausel (1982). It is seen that soil inhomogeneity alters the frequencies where

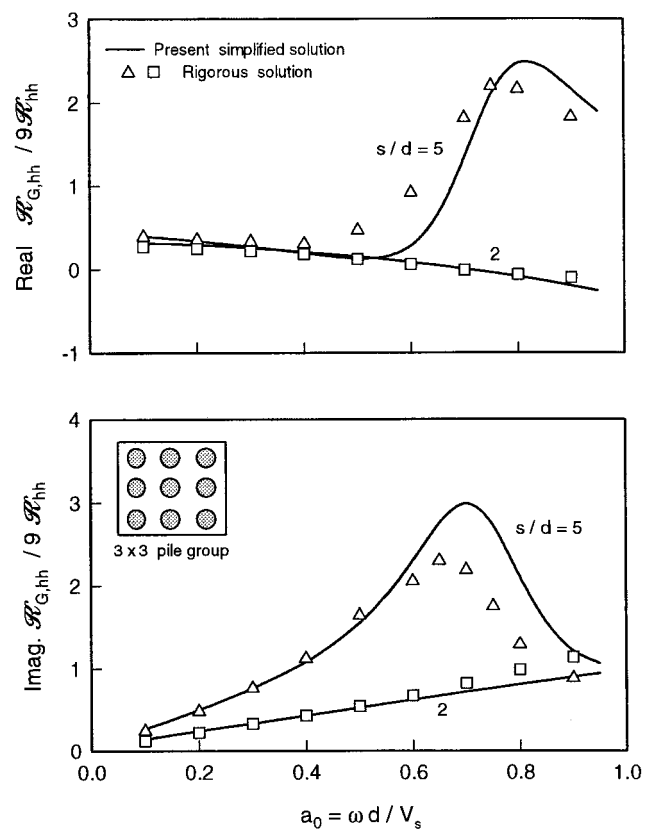


FIG. 13. Lateral Dynamic Stiffness and Damping of 3×3 Pile Group in Homogeneous Half-Space ($L/d = 20$; $E_p/E_s = 1,000$; $\nu_s = 0.4$; $\rho_p/\rho_s = 1.5$; $\beta_s = 5\%$). Comparison of Proposed Method with the Rigorous Method of Kaynia and Kausel (1982)

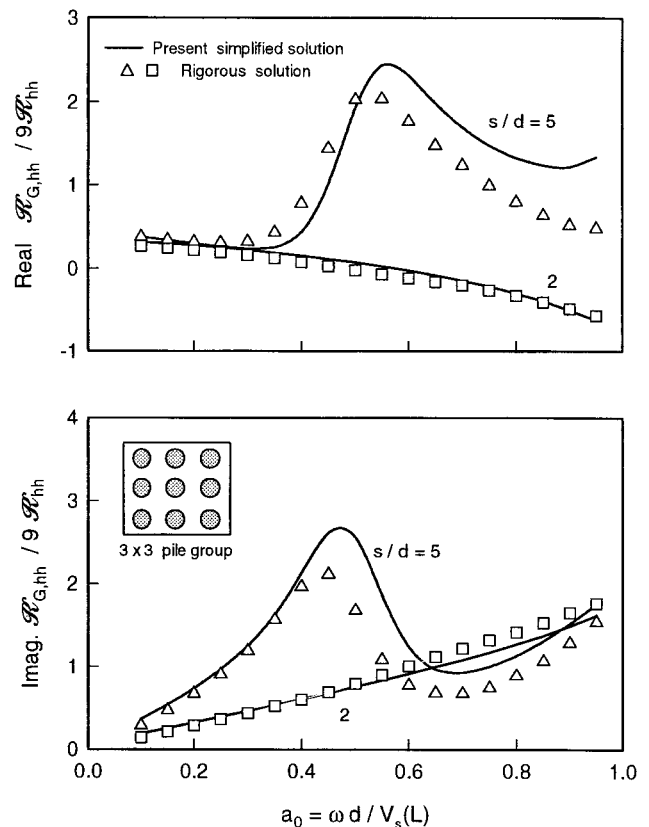


FIG. 14. Lateral Dynamic Stiffness and Damping of 3×3 Pile Group in Inhomogeneous Half-Space with Linearly Increasing Young's Modulus, from $E_s = E_p/1,250$ at Surface to $E_s = E_p/500$ at Pile Tip ($L/d = 20$; $\nu_s = 0.4$; $\rho_p/\rho_s = 1.5$; $\beta_s = 5\%$). Comparison of Proposed Method with the Rigorous Method of Kaynia and Kausel (1982)

the peaks occur. The proposed method is in convincing agreement with the more rigorous method.

ADDITIONAL INTERNAL FORCES ALONG PILES

As already mentioned, in the superposition method each pile plays two roles: the role of a source pile carrying only the head loading and the role of a receiver pile subjected to the displacement field generated by the neighboring source piles. Accordingly, the displacement and the internal forces of a pile in a group must be calculated by superimposing the force and displacement profiles corresponding to these two roles. Presently used superposition solutions properly account for the superposition of displacements (i.e., by considering the relevant interaction factors). However, for the internal forces superposition solutions traditionally consider only the forces developing on the source piles. The additional forces (hereafter called additional pile distress) developing along the receiver piles are overlooked. Despite the fact that in rigorous numerical formulations this additional distress is accounted for implicitly, its potential importance remains unexplored.

The proposed method offers a simple way for determining these additional forces. For an arbitrary pile in the group, the head load transmitted onto the pile through the cap would produce a response atop the pile that would be different than the actual response of the cap. The difference between the two responses is the (positive or negative) additional displacement (and rotation) caused by pile-to-pile interaction, which is the “receiver” response of that pile. Accordingly, the following response vector can be defined:

$$\{\delta D\}_i = \{D_G\} - [\mathcal{K}]^{-1}\{P\}_i \quad (22)$$

where $\{\delta G\}_i = (2 \times 1)$ receiver response vector; $\{D_G\}$ = response of the cap; $[\mathcal{K}]$ = impedance matrix of the single pile; $\{P\}_i$ = load atop the pile; and i = pile number. The additional response $\{\delta D\}_i$ and the zero-load condition atop the receiver pile are the required boundary conditions for determining the additional force and displacement profiles along the pile. To this end, (16) is written in the following transfer-matrix form:

$$\begin{Bmatrix} \{\delta D(z)\}_i \\ \{\delta P(z)\}_i \end{Bmatrix} = [L(z)] \begin{Bmatrix} \{\delta D\}_i \\ \{0\} \end{Bmatrix} + \sum_{j=1}^{m,j \neq i} \psi_{ij}(s, \theta) [Q(z)] \begin{Bmatrix} \{D\}_j \\ \{P\}_j \end{Bmatrix} \quad (23)$$

where $[L(z)]$ and $[Q(z)] = 4 \times 4$ transfer matrices. The sum on the right side of (23) corresponds to the “forcing term” caused by presence of the $(m - 1)$ source piles in the group, while $\{0\}$ stands for the 2×1 zero vector. From (23), the response of the receiver pile i can be determined at any depth z . More details can be found in Mylonakis (1995).

Characteristic results are given in Figs. 15 and 16. Specifically, the distribution of bending moments with depth along the corner and the center pile of a 3×3 pile group is plotted in Fig. 15. The group is embedded in a homogeneous half-space and subjected to a static horizontal force. The following observations are worthy of note in this figure. First, the bending moments within the first 3–4 pile diameters from the surface (which is the range the peak bending moment occurs) are practically unaffected by the additional distress. This can be explained easily because the additional bending moment is zero at the pile head to satisfy the corresponding boundary condition. Second, the bending moment diagram shows a significant increase (of more than 100%) at approximately 5–15 diameters down the pile. The trend is more pronounced on the center pile—understandably because this pile carries smaller head load but attracts greater neighboring effects than the corner pile. Third, the effect of additional distress on the shear force diagram along the pile is rather insignificant.

The effect of frequency on the additional bending moments is illustrated graphically in Fig. 16, for the edge pile of the

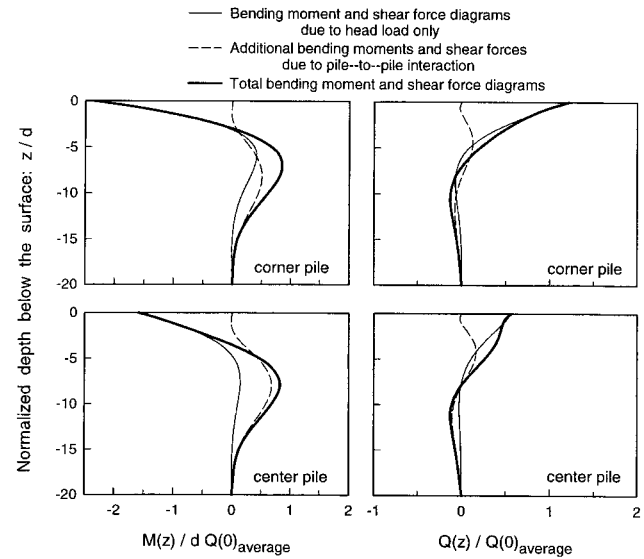


FIG. 15. Effect of Additional Traction by Neighboring Piles on Distribution of Internal Static Forces along Center and Edge Piles in 3×3 Pile Group in Homogeneous Half-Space ($L/d = 20$; $s/d = 5$; $E_p/E_s = 1,000$; $\nu_s = 0.40$; $a_0 = 0$)

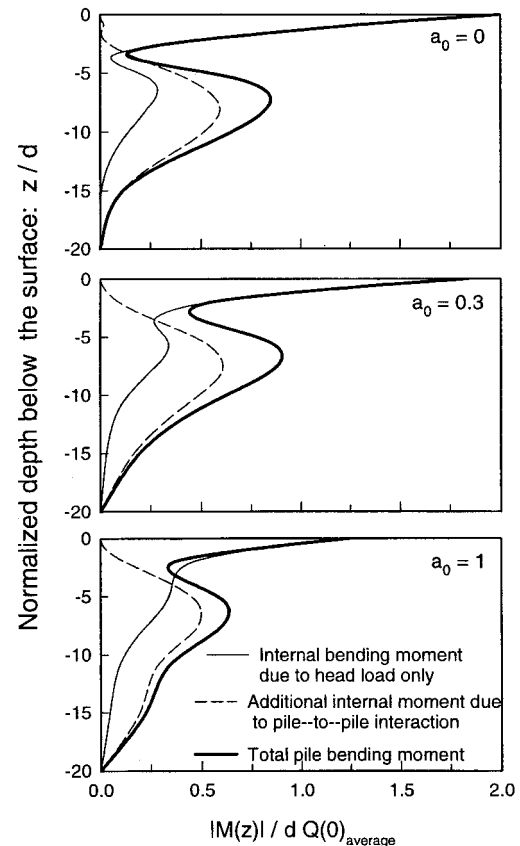


FIG. 16. Effect of Additional Bending Moment (z) Caused by Pile-to-Pile Interaction on Amplitude of Internal Bending Moment along Edge Pile of 3×3 Group of Fixed-Head Piles in Homogeneous Half-Space, for Three Different Frequencies ($E_p/E_s = 1,000$; $L/d = 20$; $\nu_s = 0.4$; $s/d = 2$)

same pile group. In this graph, the amplitude of bending moments is plotted at three different frequencies ($a_0 = 0, 0.3$, and 1). It can be seen that the bending moment transmitted to the pile from the cap depends strongly on frequency—which is a well-known result of pile-to-pile interaction (Kaynia and Kausel 1982; Dobry and Gazetas 1988). By contrast, the amplitude of the additional bending moment seems to be less sensitive to frequency. Its effect becomes more pronounced at high fre-

quencies ($a_0 = 1$) and at large depths down the pile. It should be pointed out that the maximum bending moment is always located at the top of the pile. However, the additional bending moments may become critical for the safety of the pile: (1) in the case of piles pinned at the top and (2) at large depths below the surface where the additional moments may “interfere constructively” with “kinematic” bending moments (Nikolaou et al. 1995), induced to the piles by the response of the surrounding soil to seismic waves. These effects are beyond the scope of this paper.

CONCLUSIONS AND LIMITATIONS

A simple method is presented for the dynamic response and internal forces caused by cap loading of single piles and pile groups in homogeneous, inhomogeneous, and layered soil. The method is based on a generalized dynamic Winkler model in conjunction with a three-step wave-interference solution for pile-to-pile interaction. The method permits the interaction factors between piles to be obtained in closed form and valuable insight to be gained in the physics of the problem. Dynamic interaction factors and group stiffnesses calculated with the proposed method are in convincing agreement with more rigorous solutions.

It must be understood clearly that the proposed model is limited by the assumptions of linearity for soil and pile materials and perfect bonding at the pile-soil interface. Therefore, the method may not be applicable in situations involving strong nonlinearities such as soil liquefaction. Pile response to earthquake excitation (in the form of seismic waves traveling through the soil) was not examined. Finally, the superposition principle for pile groups was assumed valid for all pile groups—an assumption that may not be of sufficient accuracy when dealing with large closely spaced pile groups or when strongly nonlinear soil effects dominate.

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